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## Towards a common origin of the elliptic flow, ridge and alignment

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### Abstract

It is claimed that elliptic flow, ridge and alignment are effects of azimuthal asymmetry, which have a common origin evolving with primary energy and stemming from the general structure of field-theoretical matrix elements. It interrelates a new ridge-phenomenon, recently found at the LHC and RHIC, with known coplanarity feature observed in collider jet physics as well as in cosmic ray studies.

Azimuthal asymmetry of multiparticle production in collisions of hadrons and nuclei was noticed in many experiments at various energies. It got the names of elliptic flow, ridge and alignment (or coplanarity).

First it was measured in azimuthal distributions of nuclear fragments at rather low energies of 2.5 GeV and 4.5 GeV per nucleon in Berkeley and Dubna (see, e.g., [1, 2]) when studying nuclei collisions with emulsion nuclei. The positive values of the second Fourier of the series expansion of fragment distributions in differences of the azimuthal angles have been obtained. Namely this parameter was later used at the RHIC and called  $v_2$  or elliptic flow:

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{dy p_T dp_T} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_r)] \right), \quad (1)$$

where  $\Psi_r$  defines the reaction plane.

The azimuthal asymmetry of three-jet events was also observed in  $Spp\bar{S}$  collider experiments [3].

In principle, several other parameters may be used to quantify the azimuthal asymmetry. In particular, in Dubna experiments the azimuthal coplanarity was described by the coefficient

$$\beta = \frac{\sum_{i>j}^n \cos 2(\phi_i - \phi_j)}{\sqrt{n(n-1)}}, \quad (2)$$

where  $\phi_i - \phi_j$  denotes the difference between azimuthal angles of particles  $i$  and  $j$ .

To describe the alignment phenomenon in cosmic ray data the coplanarity coefficient was introduced [4]:

$$\lambda_n = \frac{\sum_{i \neq j \neq k}^n \cos 2\phi_{ij}^k}{n(n-1)(n-2)}, \quad (3)$$

where  $\phi_{ij}^k$  is the angle between the straight lines connecting the  $i$ th and  $j$ th "particles" (cores) with the  $k$ th one.

In emulsion experiments, particle distributions in an individual event were often shown. Nowadays, experiments at the RHIC [5, 6, 7] and the LHC [8] provide results in a form of the two-dimensional ( $\Delta\eta$ ,  $\Delta\phi$ ) correlation plots and reveal the ridge phenomenon at large multiplicities. Since the individual events have no definite orientation in the azimuthal plane, in inclusive one-particle distributions there is no preferred direction. It appears when the trigger azimuthal angle is fixed. Thus the elongated individual events become oriented and the ridge appears. Nevertheless, it is of the same nature as elliptic flow and alignment and all the above criteria may be applied to quantify it.

The problem of origin of all these effects is widely debated now, see, e.g., [9, 10, 11]. We would like to notice that it may be ascribed to the general properties of the quantum field processes. The matrix elements of high energy processes are constructed in such a way that incoming partons become more and more coplanar with the outgoing partons [12, 13]. The key role in them is played by the propagators of exchanged partons. The matrix elements are larger where they are smaller. Considering the parton exchange in the (multi)peripheral (or multi-Regge) kinematics at very high energies, where the exchanged partons form a string (ladder) with low transferred momenta, the dominant contribution to the processes  $p_A + p_B \rightarrow k_1 + k_2 + k_3 + \dots$  with three, four, ... produced partons comes from the factors  $1/t_1 t_2$ ,  $1/t_1 t_2 t_3$ , ... [14, 15, 16, 17, 18].

For three produced partons this factor is merely

$$\frac{1}{(1 - \cos \theta_1)(1 + \cos \theta_2)}, \quad (4)$$

where  $\theta_1$  and  $\theta_2$  are polar angles to the collision axis. This factor is large if all three partons tend to be coplanar with the collision axis.

For four produced partons the factors adjacent to the ends of the graph ( $1/t_1$  and  $1/t_3$ ) are similar to the above expressions, while the factor  $1/t_2$  in between them is to be considered separately. Then the following expression appears in the denominator

$$\begin{aligned} p_A k_1 (1 - \cos \theta_1) &+ p_A k_2 (1 - \cos \theta_2) \\ &- k_1 k_2 [1 - \cos(\theta_1 - \theta_2) + \sin \theta_1 \sin \theta_2 (1 - \cos(\phi_1 - \phi_2))]. \end{aligned} \quad (5)$$

It is clearly seen that the same tendency to coplanarity is observed with small differences in polar and azimuthal angles between all the momentum vectors. Apparently, this asymptotic consideration with  $t$ -singularities should include phase space restrictions at finite energies and low- $p_T$ .

In the other words, topology of an inelastic event is basically formed by the plane, defined by two final state partons, while the other produced partons are either being collinear to them or soft.

Few comments are in order. The suggested picture assumes that not all the produced partons are resolved as (mini-)jets. Moreover, in semi-hard regime due to intrinsic momenta of partons inside of colliding hadrons, outgoing partons may form only one-side jet-like structure balanced by a back-to-back ridge. Also, the considered multi-parton final states can be related with high hadron multiplicities.

To conclude, a quantitative study of coplanarity induced by matrix elements must be performed before different models are proposed to explain the observed phenomena.

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The problem of origin of all these effects is widely debated now, see, e.g., [9]-[12]. We would like to notice that it may be ascribed to the general properties of the quantum field processes. The matrix elements of high energy processes are constructed in such a way that incoming partons become more and more coplanar with the outgoing partons [13, 14]. The key role in them is played by the propagators of exchanged partons. The matrix elements are larger where they are smaller. Considering the parton exchange in the (multi)peripheral (or multi-Regge) kinematics at very high energies, where the exchanged partons form a string (ladder) with low transferred momenta, the dominant contribution to the processes  $p_A + p_B \rightarrow k_1 + k_2 + k_3 + \dots$  with three, four, ... produced partons comes from the factors  $1/t_1 t_2$ ,  $1/t_1 t_2 t_3$ , ... [15]-[20].

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